Functions With No Zero Derivatives

A number of simple transcendental functions, such as tan(x), e^x, etc., have the property that their derivatives never vanish, no matter how many times we take the derivative, and the values of their derivatives at most rational arguments are also never zero.

Interestingly, it's possible to express some well-known problems in terms of functions that have no zero derivatives. For example, consider the simple function

$$f(x) = \frac{a}{ax+1} + \frac{b}{bx+1} + \frac{c}{cx+1}$$

where a, b, and c are (non-zero) integers, positive or negative. The non-vanishing of every derivative of this function at x=0 is equivalent to Fermat's Last Theorem, because the nth derivative is

$$n!(-1)^n \left[\left(\frac{a}{ax+1}\right)^{n+1} + \left(\frac{b}{bx+1}\right)^{n+1} + \left(\frac{c}{cx+1}\right)^{n+1} \right]$$

At x=0 this reduces to

$$n!(-1)^n \left[a^{n+1} + b^{n+1} + c^{n+1} \right]$$

which, by FLT, cannot equal zero for any integers a,b,c.

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