Proof that e is Irrational

The number e = 2.71828.. can be shown to be irrational by a very simple argument based on the power series expansion of the exponential function, which gives

$$1/e = 1/0! - 1/1! + 1/2! - 1/3! + 1/4! - ...$$

If P(k) is the kth partial sum, we see that P(k) - P(k-1) = +-1/k!, and so k((k-1)!)P(k-1) - k!P(k) = +-1. It follows that placing each pair of consecutive partial sums on a common basis, we have the relations

$$\begin{aligned} 2/6 &< 1/e < 3/6 \\ 8/24 &< 1/e < 9/24 \\ 44/120 &< 1/e < 45/120 \\ 264/720 &< 1/e < 265/720 \end{aligned}$$

and so on, where each pair of bounding numerators differs by 1, and the denominators are m!. The first of these relations proves that if 1/e is rational its denominator cannot be a divisor of 6, because then it could be written n/6 for some integer n, and there is no such integer greater than 2 and less than 3.

Similarly the next relation proves that the denominator of 1/e cannot be a divisor of 24, and the next proves that it cannot be a divisor of 120, and so on. Continuing in this way, it's clear that the denominator of 1/e cannot be a divisor of any m! for m=2,3,4,...and so on to infinity. But every integer k is a divisor of m! for all m >= k, so 1/e (and therefore e) cannot be a rational number.

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