Show that A_4 has no subgroup of order 6.

Our proof is by contradiction. Note that by this demonstration we prove that Lagrange's Theorem has no general converse.

We introduce the concept of the *order spectrum*, that is, the set of counts of elements of a group G of various orders of elements in G, and denote it by $\#G$. Thus $\#A_4 = \{1, 3, 8\}$, which accounts for all 12 elements of A_4. In other words, A_4 has 1 element of order 1, 3 of order 2, and 8 of order 3. So we already know that A_4 has no element of order 6.

Notational Notes: If H is a subgroup of G then the index of H in G is denoted as $|G : H|$. The centralizer of an element a of G is represented by $C(a)$. The subgroup generated by an element a of G is represented by $< a >$. The symbol $\equiv$ means "is isomorphic to," or "is defined to be."

Lemma 1 (Cauchy's Theorem): If p divides the order of a finite group G then G has an element of order p.

Lemma 2: If $|G : H| = 2$ for $H < G$, then H is normal in G.

Lemma 3 (Lagrange): The order of a subgroup divides the order of the group.